

# **THE COPPICE PRIMARY SCHOOL**

## **Mathematics Calculation Strategies 2021**

### **Multiplication and Division**



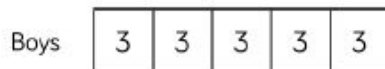
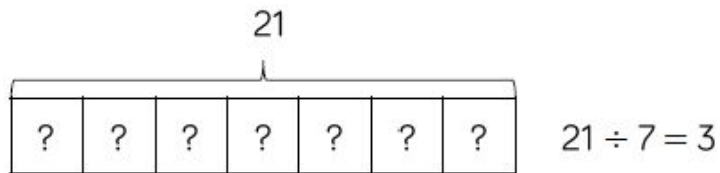
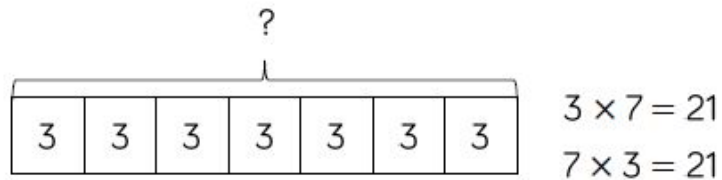
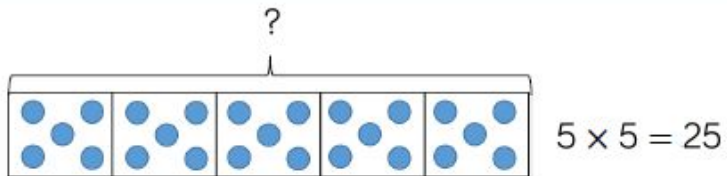
This part of maths calculation plan covers the strategies we use for teaching multiplication and division.

Each operation is broken down into skills and each skill has examples showing the models and images that could be used to effectively teach that concept.

As well supporting teachers with their planning, we hope this document will help parents with understanding the strategies we use and the way problems could be represented.



# Bar Model



## Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.



# Number Shapes



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



## Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd  $\times$  odd = even, odd  $\times$  even = odd, even  $\times$  even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.



## Bead Strings



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

## Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.



# Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

## Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

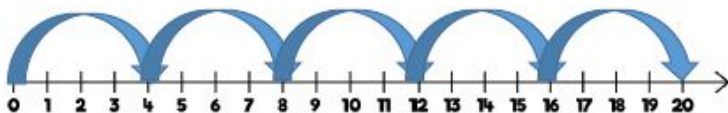
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.



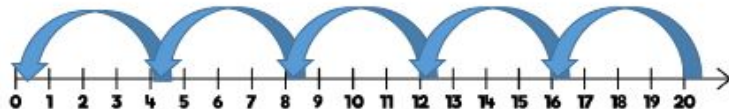


# Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

## Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

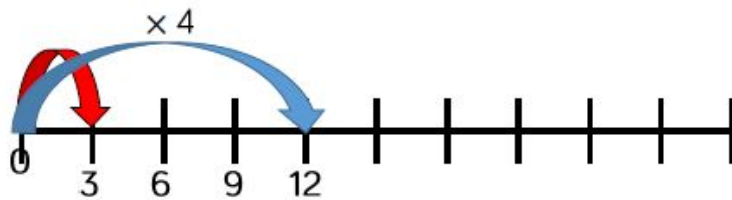
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



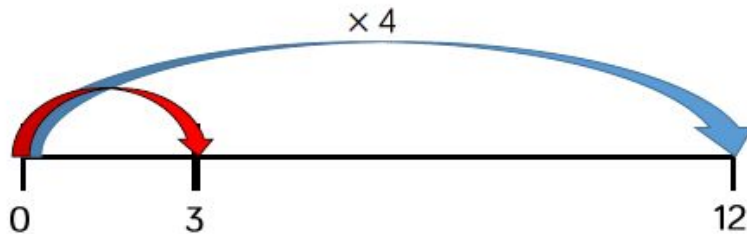
## Number Lines (blank)



A red car travels 3 miles.

A blue car 4 times further.

How far does the blue car travel?



A blue car travels 12 miles.

A red car 4 times less.

How far does the red car travel?

### Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

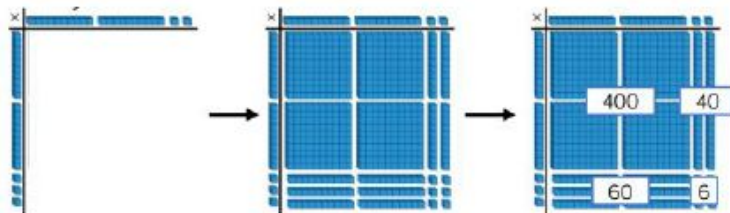




## Base 10/Dienes (multiplication)

Hundreds	Tens	Ones
		● ● ● ●
		● ● ● ●
		● ● ● ●

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline 1 \end{array}$$



### Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.



## Base 10/Dienes (division)

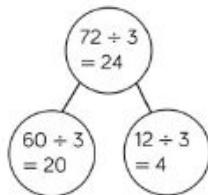


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



### Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.


$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ \hline 12 \end{array}$$
$$\begin{array}{r} 44 \\ \times 32 \\ \hline 88 \\ 880 \\ + 1320 \\ \hline 1408 \end{array}$$

## Benefits

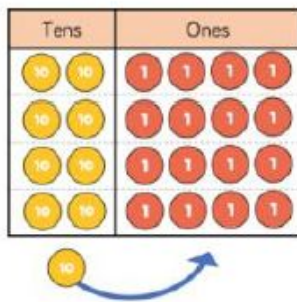
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

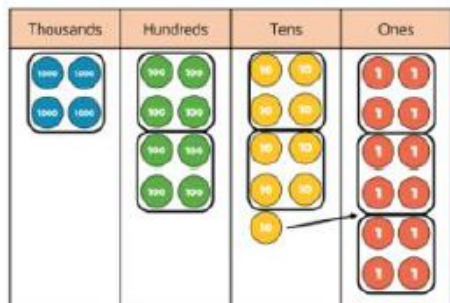
Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.



# Place Value Counters (division)



$$\begin{array}{c} 96 \div 4 \\ = 24 \\ \swarrow \quad \searrow \\ 80 \div 4 = 20 \quad 16 \div 4 = 4 \end{array}$$



$$\begin{array}{r} 1223 \\ 4 \overline{) 4892} \end{array}$$

## Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

MULTIPLICATION





Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Short written method Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method
Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method

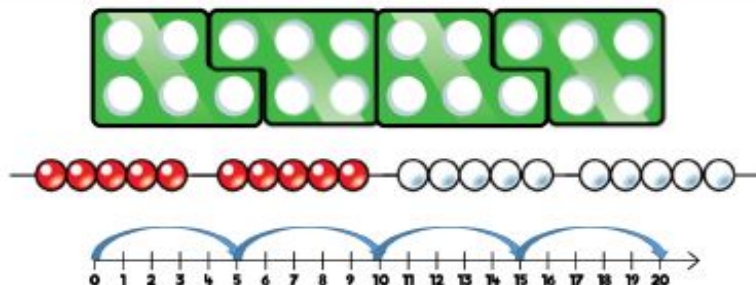
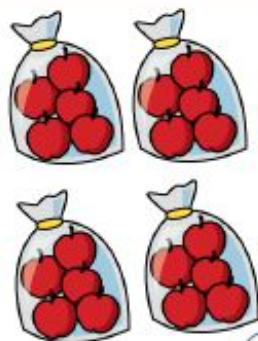
Skill	Year	Representations and models	
Multiply 2-digit by 2-digit numbers	5	Place value counters Base 10	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	



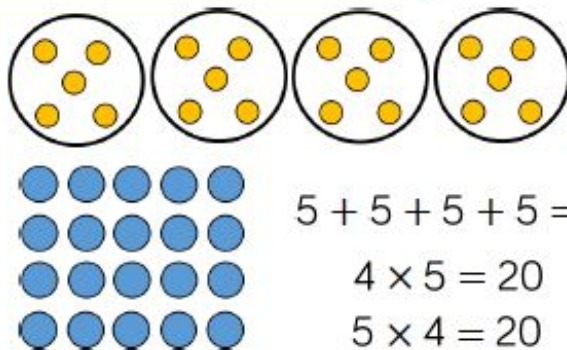
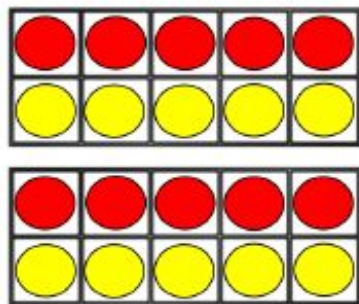


## Skill: Solve 1-step problems using multiplication

Year: 1/2



One bag holds 5 apples.  
How many apples do 4 bags hold?



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

Children represent multiplication as repeated addition in many different ways.

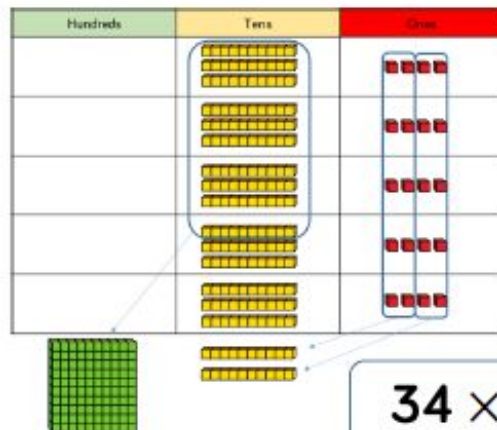
In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

In Year 2, children are introduced to the multiplication symbol.



## Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4



$$34 \times 5 = 170$$

	H	T	O
		3	4
x			5
	1	7	0
	1	2	

	H	T	O		
		3	4		
x			5		
		2	0	(5 x 4)	
+	1	5	0	(5 x 30)	
	1	7	0		



Teachers may decide to first look at the expanded column method before moving on to the short multiplication method.

The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.



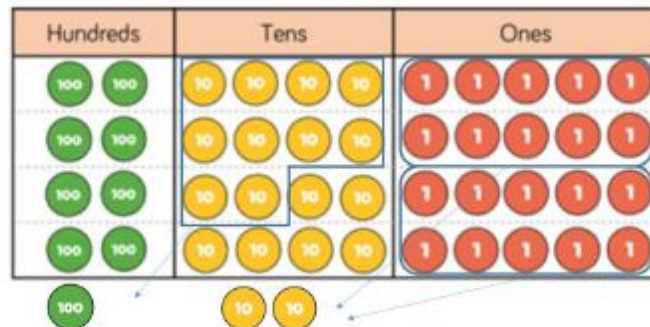
## Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 3/4



	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

$$245 \times 4 = 980$$



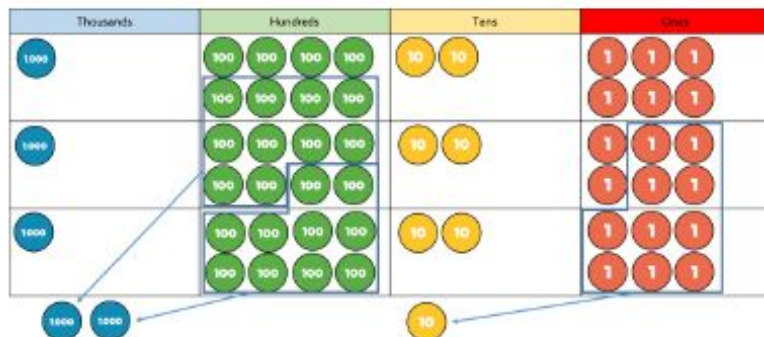
When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.

Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.



## Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



$$1,826 \times 3 = 5,478$$

	Th	H	T	O
	1	8	2	6
$\times$				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.

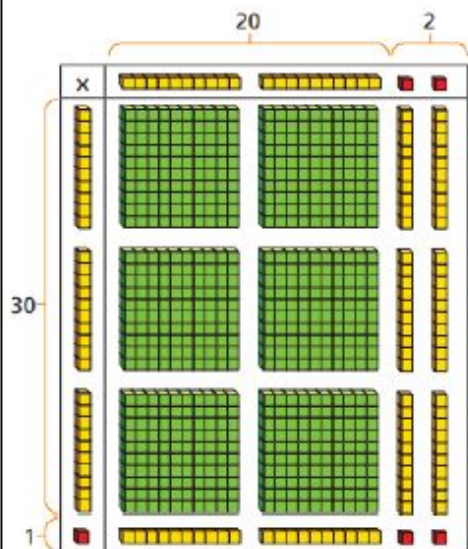
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.





## Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5



×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

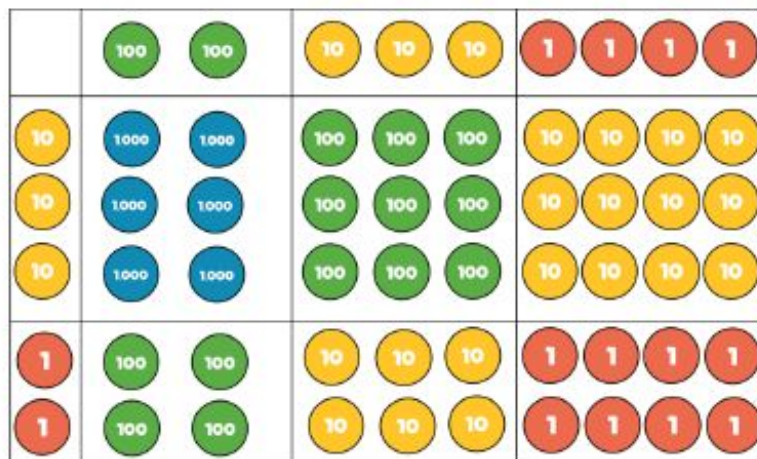
$$22 \times 31 = 682$$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.



# Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5



Th	H	T	O
	2	3	4
x		3	2
	4	6	8
17	10	2	0
7	4	8	8

x	200	30	4
30	6,000	900	120
2	400	60	8

$$234 \times 32 = 7,488$$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.





## Skill: Multiply 4-digit numbers by 2-digit numbers

Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2	1	9	1	2
2	5	3	7	
5	4	7	8	0
1		1		
7	6	6	9	2

1

$$2,739 \times 28 = 76,692$$

When multiplying 4-digits by 2-digits, children should be confident in the written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

DIVISION



Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model

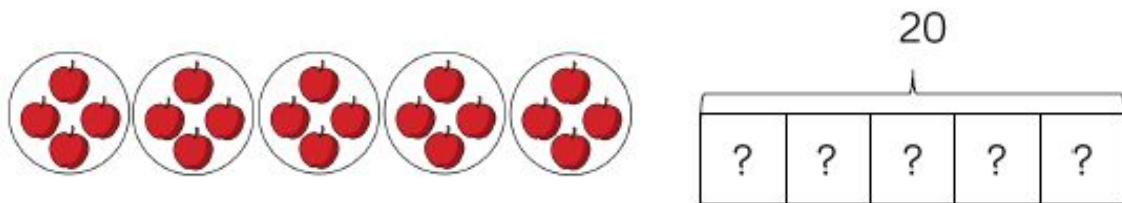
Skill	Year	Representations and models	
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

Skill	Year	Representations and models	
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

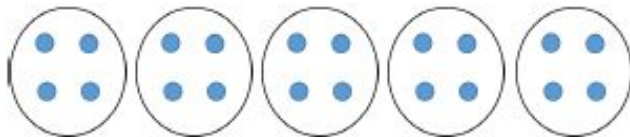
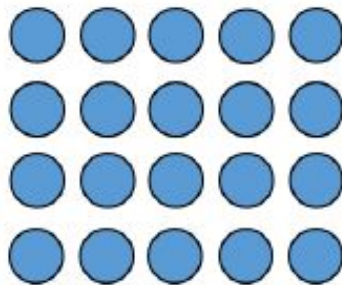


## Skill: Solve 1-step problems using multiplication (sharing)

Year: 1/2



There are 20 apples altogether.  
They are shared equally between 5 bags.  
How many apples are in each bag?



$$20 \div 5 = 4$$

Children solve problems by sharing amounts into equal groups.

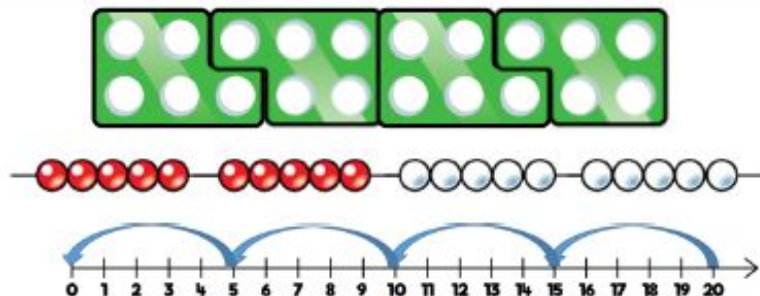
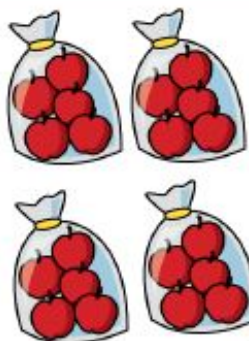
In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

In Year 2, children are introduced to the division symbol.

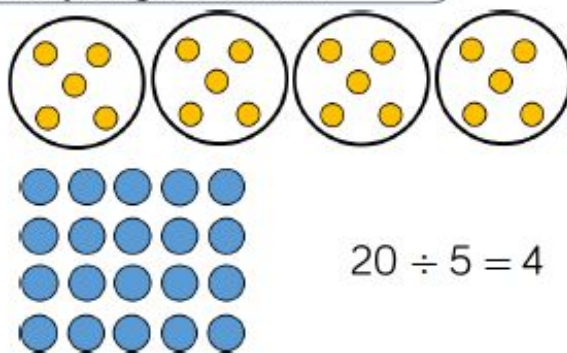
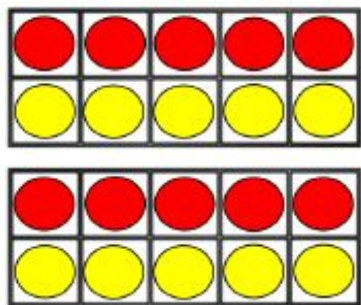


## Skill: Solve 1-step problems using division (grouping)

Year: 1/2



There are 20 apples altogether.  
They are put in bags of 5.  
How many bags are there?



$$20 \div 5 = 4$$

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

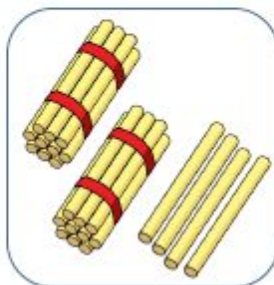
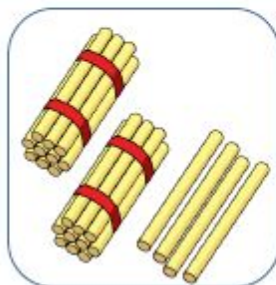




## Skill: Divide 2-digits by 1-digit (sharing with no exchange)

Year: 1/2

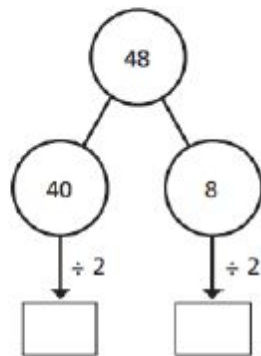
Tens	Ones
 	 
 	 



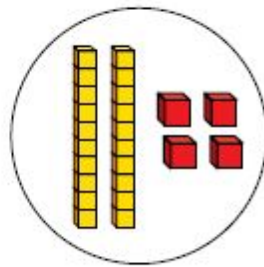
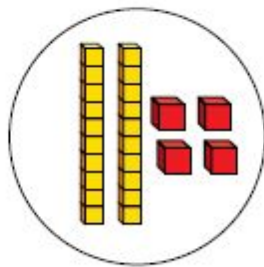
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.



$$48 \div 2 = 24$$














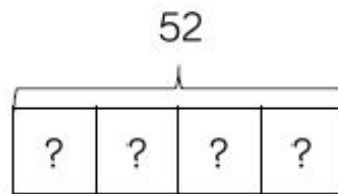


## Skill: Divide 2-digits by 1-digit (sharing with exchange)

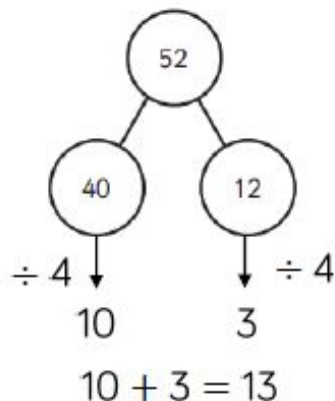
Year: 3/4



Tens	Ones
	
	
	
	



$$52 \div 4 = 13$$




Tens	Ones
	
	
	
	

When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.

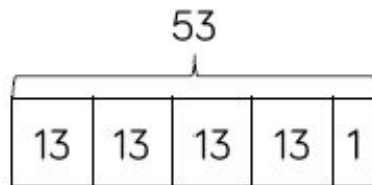
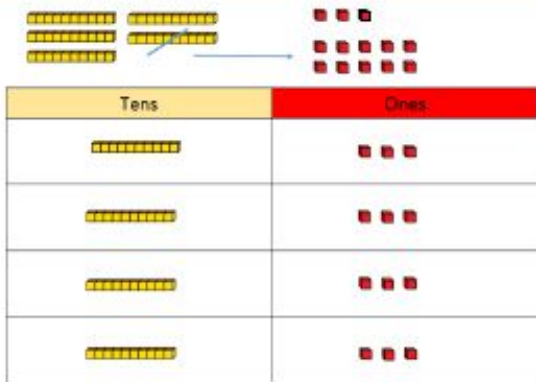
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

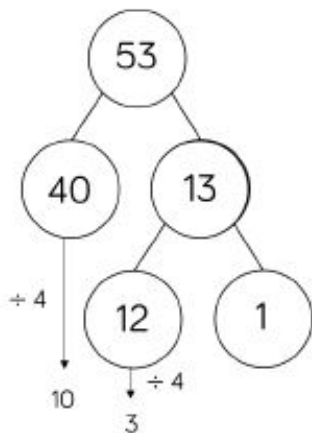


## Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4



$$53 \div 4 = 13 \text{ r}1$$

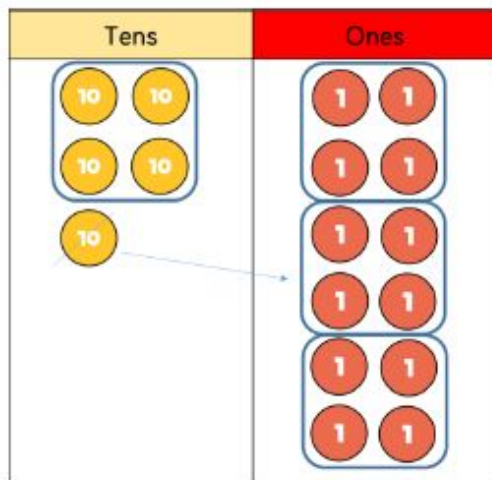


When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.



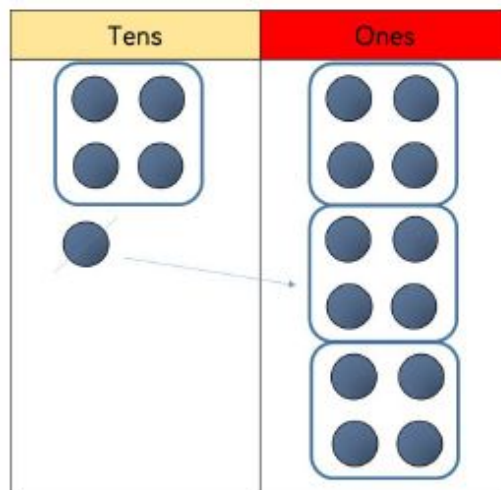
## Skill: Divide 2-digits by 1-digit (grouping)

Year: 4/5



$$52 \div 4 = 13$$

		1	3	
	4	5	12	



When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

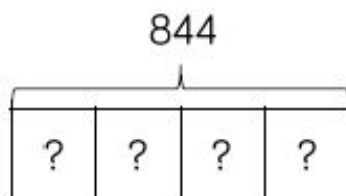
Remainders can also be seen as they are left ungrouped.



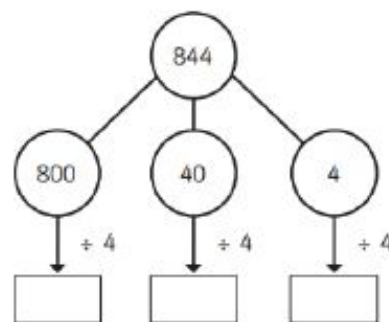
## Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

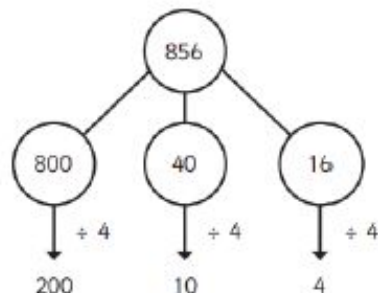
$$844 \div 4 = 122$$



H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1



$$844 \div 4 = 122$$



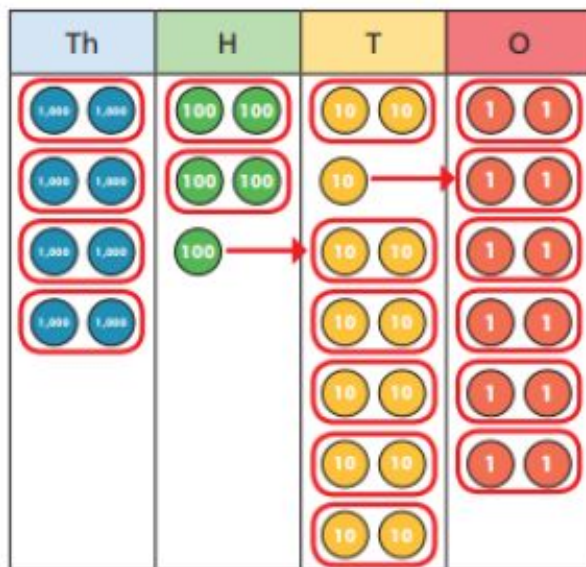
Hundreds	Tens	Ones
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1

Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.



## Skill: Divide 4-digits by 1-digit (grouping)

Year: 5



	4	2	6	6
2	8	5	13	12

$$8,532 \div 2 = 4,266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit.

Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.





## Skill: Divide multi digits by 2-digits (short division)

Year: 6

		0	3	6
	12	4	<sup>4</sup> 3	<sup>7</sup> 2

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	<sup>7</sup> 3	<sup>13</sup> 3	<sup>13</sup> 5

15	30	45	60	75	90	105	120	135	150
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When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.





## Skill: Divide multi-digits by 2-digits (long division)

Year: 6

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

$$\begin{array}{l} \cdot 12 \times 1 = 12 \\ 12 \times 2 = 24 \\ 12 \times 3 = 36 \\ 12 \times 4 = 48 \\ 12 \times 5 = 60 \\ 12 \times 6 = 72 \\ 12 \times 7 = 84 \\ 12 \times 8 = 96 \\ 12 \times 9 = 108 \\ 12 \times 10 = 120 \end{array}$$

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	3	3	5
-	6	0	0	0
	1	3	3	5
-	1	2	0	0
		1	3	5
-		1	3	5
				0

$$\begin{array}{l} 1 \times 15 = 15 \\ 2 \times 15 = 30 \\ 3 \times 15 = 45 \\ 4 \times 15 = 60 \\ 5 \times 15 = 75 \\ 10 \times 15 = 150 \end{array}$$

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.



## Skill: Divide multi digits by 2-digits (long division)

Year: 6

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

- $1 \times 15 = 15$
- $2 \times 15 = 30$
- $3 \times 15 = 45$
- $4 \times 15 = 60$
- $5 \times 15 = 75$
- $10 \times 15 = 150$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0		
			7	2	
	–		6	0	
			1	2	

$$372 \div 15 = 24\frac{4}{5}$$



## Glossary

**Array** – An ordered collection of counters, cubes or other item in rows and columns.

**Commutative** – Numbers can be multiplied in any order.

**Dividend** – In division, the number that is divided.

**Divisor** – In division, the number by which another is divided.

**Exchange** – Change a number or expression for another of an equal value.

**Factor** – A number that multiplies with another to make a product.

**Multiplicand** – In multiplication, a number to be multiplied by another.

**Partitioning** – Splitting a number into its component parts.

**Product** – The result of multiplying one number by another.

**Quotient** – The result of a division

**Remainder** – The amount left over after a division when the divisor is not a factor of the dividend.

**Scaling** – Enlarging or reducing a number by a given amount, called the scale factor